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Counterflow double-pipe heat exchanger analysis using a mixed lumped–differential formulation

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Abstract—An analysis is made of double-pipe heat exchangers under a thermally developing countercurrent flow condition. A lumped–differential mixed formulation is employed, by radially lumping the temperature field in the outer channel, which results in a more involved boundary condition for the inner differential system, involving the axially varying outer channel bulk temperature. The generalized integral transform technique is utilized to provide a reliable and straightforward analytical solution to this class of problems. Numerical results for heat transfer quantities are presented in terms of the dimensionless governing parameters along the thermal entry region, allowing for critical comparisons against the concurrent flow situation, limiting solutions and engineering-type correlations.

INTRODUCTION

DOUBLE-PIPE heat exchangers are commonly used devices in thermal engineering practice, especially in connection with relatively small exchange area requirements, pressurized systems and laboratory set-ups [1], due to their simple construction and maintenance characteristics. Stein [2, 3] appears to be the first investigator to formulate such a class of problems and attempt an analytical solution to the two governing energy equations coupled at the boundary conditions. For the concurrent laminar flow situation, under a hydrodynamically fully developed condition, a number of contributions followed [4–7] that employed eigenfunction expansion-type approaches and, particularly after the advancement of an automatic and reliable solution technique for the related eigenvalue problem [7], filled the gap between Stein's pioneering work and the present need for accurate reference results in heat exchanger design. For counterflow situations, however, the utilization of such approaches is not a trivial matter, due to the more involved auxiliary eigenvalue problem which results from the coupled nature between inlet and exit stream temperatures, as demonstrated through the efforts of Nunge and Gill [8]. Therefore, an alternative simplified formulation was proposed by Stein and Sastri [9], which radially lumps the temperature distribution in the outer annular channel and maintains the partial differential formulation for the inner tube. As a result, the interface condition incorporates the bulk temperature of the outer channel, providing a more general boundary condition for the single partial differential equation that governs the inner tube temperature distribution. An approximate solution for this formulation was then obtained, based on the Laplace transform technique, and applied in subsequent developments [10, 11] for both concurrent and countercurrent flow configurations. It was not until quite recently [12] that applicability limits for this simplified formulation were established, in terms of the governing dimensionless parameters, through a critical comparison with the benchmark results available for concurrent flow [7] along the thermal entry region. Also in ref. [12], the exact analytical solution for the mixed lumped–differential formulation of concurrent flow double-pipe heat exchangers is provided, by extending the ideas in the generalized integral transform technique [13–19], as applied to a priori non-transformable diffusion/convection problems. In the present contribution, the formalism in ref. [12] is extended for the counterflow situation, allowing for the establishment of heat transfer results in a wide range of governing parameters such as exchanger length, heat capacity flow rate ratio, relative thermal resistance of the interface and dimensionless axial coordinate. Numerical results for the quantities of practical interest are then presented, including bulk temperatures, Nusselt numbers and exchanger effectiveness. Since a very limited number of works were previously concerned with countercurrent flow, we take advantage of the present reliable approach to examine more carefully different aspects related to this important configuration, such as comparisons with concurrent flow results, with limiting situations of prescribed wall temperature and heat flux, and with an engineering-type correlation for the heat transfer coefficient in double-pipe heat exchangers [20].

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NOMENCLATURE

a_k inner and outer tube radii, $k = 1, 2$
b interfacial wall thickness
c_k specific heats of inner and outer fluids, $k = 1, 2$
k_k thermal conductivities of inner and outer fluids, $k = 1, 2$
L^* exchanger length, dimensional
$\dot{m}_k$ mass flow rates, $k = 1, 2$
R dimensionless radial coordinate
T_k temperature distributions, dimensional, $k = 1, 2$
T_{ek} inlet temperatures, dimensional, $k = 1, 2$
u_k velocity profiles, dimensional, $k = 1, 2$
$\bar{u}_k$ average velocities, $k = 1, 2$
$U_k(R)$ velocity profiles, dimensionless, $k = 1, 2$
x or y transversal coordinate, dimensional
z axial coordinate, dimensional
Z axial coordinate, dimensionless.

Greek symbols
α_k thermal diffusivities, $k = 1, 2$
$\theta_k(R, Z)$ temperature distributions, dimensionless, $k = 1, 2$
$\theta_{av,k}(Z)$ bulk temperatures, dimensionless, $k = 1, 2$
λ eigenvalues of matrix A, equation (14b)
μ eigenvalues of Sturm–Liouville problem, equations (7)
ρ_k densities, $k = 1, 2$.

Subscripts
av average
1 inner stream
2 outer stream
w interfacial wall
i, j order of eigenvalue.

Superscript
$-$ integral transformed quantity.

ANALYSIS

In order to demonstrate the lumping procedure at the outer annular channel, we start from the fully differential formulation for a double-pipe heat exchanger in dimensionless form [7, 12], written for a counterflow configuration. Laminar flow is assumed at both tube and shell sides for derivation purposes, but the formulation to be obtained is directly applicable to turbulent flow at the annular region. According to the coordinate system presented in Fig. 1, the problem formulation, assuming constant physical properties and hydrodynamically fully developed flow, is given by:

Inner tube

$$RU_1(R)\frac{\partial\theta_1(R,Z)}{\partial Z} = \frac{\partial}{\partial R}\left[R\frac{\partial\theta_1(R,Z)}{\partial R}\right],$$
$$0 < R < 1, \quad 0 < Z \leqslant L \quad \text{(1a)}$$

Outer annular channel

$$-\frac{KH}{2}[1-R(1-R^*)]U_2(R)\frac{\partial\theta_2(R,Z)}{\partial Z} = \frac{\partial}{\partial R}\left[(1-R(1-R^*))\frac{\partial\theta_2(R,Z)}{\partial R}\right],$$
$$0 < R < 1, \quad 0 \leqslant Z < L \quad \text{(1b)}$$

with inlet conditions at opposite ends

$$\theta_1(R,0) = 0, \quad \theta_2(R,L) = 1, \quad 0 \leqslant R \leqslant 1 \quad \text{(1c, d)}$$

and boundary conditions

$$\frac{\partial\theta_1(0,Z)}{\partial R} = 0, \quad \frac{\partial\theta_2(0,Z)}{\partial R} = 0, \quad 0 \leqslant Z \leqslant L \quad \text{(1e, f)}$$

$$K\frac{\partial\theta_1(1,Z)}{\partial R} + \frac{\partial\theta_2(1,Z)}{\partial R} = 0, \quad 0 \leqslant Z \leqslant L \quad \text{(1g)}$$

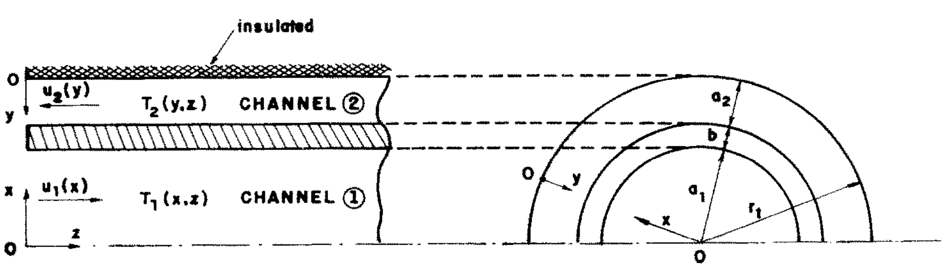

FIG. 1. Geometry and coordinate system for double-pipe heat exchanger analysis.

$$K_w \frac{\partial \theta_1(1,Z)}{\partial R} + \theta_1(1,Z) - \theta_2(1,Z) = 0, \quad 0 \leqslant Z \leqslant L \tag{1h}$$

where various dimensionless groups are given by

$R = x/a_1$ or y/a_2, dimensionless transversal coordinate

$Z = \alpha_1 z/(\bar{u}_1 a_1^2)$, dimensionless axial coordinate

$U_i(R) = u_i/\bar{u}_i$, $i = 1, 2$, dimensionless velocity profiles

$L = \alpha_1 L^*/\bar{u}_1 a_1^2$, dimensionless exchanger length

$\theta_i(R,Z) = (T_i - T_{e_1})/(T_{e_2} - T_{e_1})$, $i = 1, 2$ dimensionless temperature profiles

$R^* = (a_1 + b)/(a_1 + b + a_2)$, aspect ratio

$K = (k_1/k_2)(a_1/(a_1+b))(a_2/a_1)$, relative thermal resistance of fluids

$K_w = (k_1/k_w) \ln(1 + b/a_1)$, relative thermal resistance of wall

$$H = \dot{m}_2 C_2/\dot{m}_1 C_1 = 2(\rho_2 c_2 \bar{u}_2)/(\rho_1 c_1 \bar{u}_1)\cdot(a_2/a_1)\cdot((a_1+b)/a_1),$$

heat capacity flow rate ratio for the limiting case of $R^* \to 1$. (2)

The dimensionless velocity profile of the inner stream is obtained from

$$U_1(R) = 2(1 - R^2) \tag{3}$$

while the respective expression for the outer stream [7], either for laminar or turbulent flow conditions, is not in fact required within the present mixed lumped–differential formulation.

The relative merits and limitations of the above classical model are discussed in the earlier works of Stein [2, 3]. A lumped formulation for the temperature distribution in the outer channel was first proposed in ref. [9], by integrating equation (1b) over the cross-section $0 < R < 1$ and utilizing equation (1g) to yield the following ordinary differential equation for the bulk temperature in the annulus:

$$\frac{d\theta_{2,av}(Z)}{dZ} - \frac{2}{H^*}\frac{\partial \theta_1(1,Z)}{\partial R} = 0, \quad 0 \leqslant Z < L \tag{4a}$$

where the heat capacity flow rate ratio is given by

$$H^* = \frac{\dot{m}_2 C_2}{\dot{m}_1 C_1} = H\left[1 + \frac{1-R^*}{2R^*}\right] \tag{4b}$$

with the inlet condition

$$\theta_{2,av}(L) = 1. \tag{4c}$$

The problem formulation for the inner stream becomes

$$RU_1(R)\frac{\partial \theta_1(R,Z)}{\partial Z} = \frac{\partial}{\partial R}\left[R\frac{\partial \theta_1(R,Z)}{\partial R}\right],$$
$$0 < R < 1, \quad 0 < Z \leqslant L \tag{5a}$$

with inlet and boundary conditions, respectively

$$\theta_1(R,0) = 0, \quad 0 \leqslant R \leqslant 1 \tag{5b}$$

$$\frac{\partial \theta_1(0,Z)}{\partial R} = 0, \quad 0 < Z \leqslant L \tag{5c}$$

$$K_w \frac{\partial \theta_1(1,Z)}{\partial R} + \theta_1(1,Z) = \theta_{2,av}(Z), \quad 0 < Z \leqslant L \tag{5d}$$

where in equation (5d) the basic assumption of the lumping procedure was employed by letting $\theta_2(1,Z) \simeq \theta_{2,av}(Z)$, which corresponds to admitting that temperature gradients are not so significant in the radial direction within the annular region. Clearly, the parameter that measures the relative thermal resistances of the two streams, K, cancels out and, as shown in ref. [12], the present formulation becomes increasingly accurate for decreasing values of K.

Once a solution for system (4), (5) has been obtained, quantities of practical interest can be computed from their definitions [7, 12].

Average fluid temperature at the inner tube

$$\theta_{1,av}(Z) = \frac{\int_0^1 W_1(R)\theta_1(R,Z)\,dR}{\int_0^1 W_1(R)\,dR} \tag{6a}$$

where

$$W_1(R) = RU_1(R). \tag{6b}$$

Local Nusselt number at the inner tube

$$Nu_1(Z) = \frac{2\dfrac{\partial \theta_1(1,Z)}{\partial R}}{\theta_1(1,Z) - \theta_{1,av}(Z)}. \tag{6c}$$

Overall Nusselt number

$$\frac{1}{Nu_0(Z)} = \frac{1}{Nu_1(Z)} + \frac{K_w}{2} = \left\{\frac{2\dfrac{\partial \theta_1(1,Z)}{\partial R}}{\theta_{2,av}(Z) - \theta_{1,av}(Z)}\right\}^{-1}. \tag{6d, e}$$

Heat exchanger effectiveness

$$\varepsilon(Z) = \frac{Q(Z)}{Q_{max}} \tag{6f}$$

or

$$\varepsilon(Z) = \theta_{1,av}(Z), \quad \text{for } H^* \geqslant 1 \tag{6g}$$

$$\varepsilon(Z) = \frac{\theta_{1,av}(Z)}{H^*}, \quad \text{for } H^* < 1 \tag{6h}$$

where average fluid temperatures are related by

$$\theta_{2,\mathrm{av}}(Z) = 1+\frac{1}{H^*}[\theta_{1,\mathrm{av}}(Z)-\theta_{1,\mathrm{av}}(L)]. \quad (6i)$$

Average overall Nusselt number

$$\overline{Nu_0}(L) = \frac{1}{L}\int_0^L Nu_0(Z)\,\mathrm{d}Z \quad (6j)$$

or

$$\overline{Nu_0}(L) = \frac{1}{L}\frac{H^*}{1-H^*}\ln\left[\frac{1-\theta_{1,\mathrm{av}}(L)}{1-\frac{1}{H^*}\theta_{1,\mathrm{av}}(L)}\right] \quad (6k)$$

and, for $H^* = 1$

$$\overline{Nu_0}(L) = \frac{1}{L}\frac{\theta_{1,\mathrm{av}}(L)}{1-\theta_{1,\mathrm{av}}(L)}. \quad (6l)$$

The ideas in the generalized integral transform technique [13–19] are now utilized to provide an analytical solution to equations (5a)–(5d), with the coupling equations for the outer stream bulk temperature, equations (4a), (4c). Following the formalism in ref. [12], the appropriate Graetz-type auxiliary eigenvalue problem is taken as:

$$\frac{\mathrm{d}}{\mathrm{d}R}\left[R\frac{\mathrm{d}\psi_i(R)}{\mathrm{d}R}\right]+\mu_i^2 RU_1(R)\psi_i(R) = 0, \quad 0 < R < 1 \quad (7a)$$

$$\frac{\mathrm{d}\psi_i(0)}{\mathrm{d}R} = 0; \quad K_w\frac{\mathrm{d}\psi_i(1)}{\mathrm{d}R}+\psi_i(1) = 0 \quad (7b,c)$$

which can be accurately and automatically solved according to previous developments [12, 15], and allows definition of the following integral transform pair:

$$\bar{\theta}_{1,i}(Z) = \frac{1}{N_i^{1/2}}\int_0^1 RU_1(R)\psi_i(R)\theta_1(R,Z)\,\mathrm{d}R, \quad \text{transform} \quad (8a)$$

$$\theta_1(R,Z) = \sum_{i=1}^{\infty}\frac{1}{N_i^{1/2}}\psi_i(R)\bar{\theta}_{1,i}(Z), \quad \text{inversion} \quad (8b)$$

where the norm is given by

$$N_i = \int_0^1 RU_1(R)\psi_i^2(R)\,\mathrm{d}R. \quad (9)$$

Equation (5a) is now operated on with

$$\frac{1}{N_i^{1/2}}\int_0^1 \psi_i(R)\,\mathrm{d}R$$

and problem (7) is employed to yield, after utilizing the boundary conditions at $R = 1$

$$\frac{\mathrm{d}\bar{\theta}_{1,i}(Z)}{\mathrm{d}Z}+\mu_i^2\bar{\theta}_{1,i}(Z) = -\frac{1}{N_i^{1/2}}\frac{\mathrm{d}\psi_i(1)}{\mathrm{d}R}\theta_{2,\mathrm{av}}(Z), \quad i = 1,2,\ldots. \quad (10)$$

Equation (10) represents an infinite system of ordinary differential equations for the transformed potentials, $\bar{\theta}_{1,i}(Z)$, coupled with equations (4) through the average temperature, $\theta_{2,\mathrm{av}}$. Equation (4a) is now rewritten in terms of the transformed potentials to establish the final relation with equation (10). Therefore, an alternative expression for the derivative $\partial\theta_1(1,Z)/\partial R$ is obtained by integrating equation (5a) in the cross-section $0 < R < 1$, to provide

$$\frac{\partial\theta_1(1,Z)}{\partial R} = \sum_{j=1}^{\infty}\bar{f}_j\frac{\mathrm{d}\bar{\theta}_{1,j}(Z)}{\mathrm{d}Z} \quad (11a)$$

where

$$\bar{f}_j = \frac{1}{N_j^{1/2}}\int_0^1 RU_1(R)\psi_j(R)\,\mathrm{d}R. \quad (11b)$$

Equation (11a) is substituted back into equation (4a) and equation (10) is utilized to produce the final coupled system in normal form

$$\frac{\mathrm{d}\bar{\theta}_{1,i}(Z)}{\mathrm{d}Z} = -\mu_i^2\bar{\theta}_{1,i}(Z)+\mu_i^2\bar{f}_i\theta_{2,\mathrm{av}}(Z), \quad i = 1,2,\ldots \quad (12a)$$

$$\frac{\mathrm{d}\bar{\theta}_{2,\mathrm{av}}(Z)}{\mathrm{d}Z} = \sum_{j=1}^{\infty}B_j\bar{\theta}_{1,j}(Z)+\frac{2}{H^*}E\theta_{2,\mathrm{av}}(Z) \quad (12b)$$

with inlet conditions

$$\bar{\theta}_{1,i}(0) = 0, \quad i = 1,2,\ldots \quad (12c)$$

$$\theta_{2,\mathrm{av}}(L) = 1 \quad (12d)$$

where

$$B_j = -\frac{2}{H^*}\mu_j^2\bar{f}_j; \quad E = \sum_{j=1}^{\infty}\mu_j^2\bar{f}_j. \quad (12e,f)$$

For computational purposes, the denumerable system (12) is truncated to a sufficiently large finite order, N, and rewritten in matrix form as

$$\mathbf{y}'(Z) = A\mathbf{y}(Z), \quad 0 < Z < L \quad (13a)$$

$$y_i(0) = 0, \quad i = 1,2,\ldots,N \quad (13b)$$

$$y_{N+1}(L) = 1 \quad (13c)$$

where

$$\mathbf{y}(z) = \{\bar{\theta}_{1,1}(Z),\bar{\theta}_{1,2}(Z),\ldots,\bar{\theta}_{1,N}(Z),\theta_{2,\mathrm{av}}(Z)\}^T \quad (13d)$$

$$A = \{A_{ij}\}, \quad A_{ij} = \begin{cases} -\delta_{ij}\mu_i^2, & i,j \leqslant N \\ \mu_i^2\bar{f}_i, & i \leqslant N, j = N+1 \\ B_j, & i = N+1, j \leqslant N \\ \frac{2}{H^*}E, & i = N+1, j = N+1. \end{cases} \quad (13e)$$

System (13) can be solved analytically in the form

$$y(Z) = \sum_{j=1}^{N+1} c_j \zeta^{(j)} e^{\lambda_j Z} \tag{14a}$$

where the matrix eigenvalues, λ_j, and corresponding eigenvectors, $\zeta^{(j)}$, are obtained from the matrix eigenvalue problem

$$(A-\lambda I)\zeta = 0 \tag{14b}$$

and the constants, c_j, are determined from satisfaction of the inlet conditions, equations (12c), (12d), which results in the following algebraic problem:

$$\sum_{j=1}^{N+1} c_j \zeta_i^{(j)} = 0, \quad i = 1,2,\ldots,N$$

$$\sum_{j=1}^{N+1} c_j \zeta_{N+1}^{(j)} e^{\lambda_j L} = 1. \tag{14c}$$

Problems (14b), (14c) can be readily and accurately solved through well established subroutines available in scientific subroutine packages, such as the IMSL library [21]. Alternatively, system (13) can be directly solved by a boundary value problem solver such as the routine DVCPR in the IMSL package. Once the transformed potentials have been obtained, the inversion formula (8b) is recalled to provide the complete temperature distribution within the inner tube, while the last component of the solution vector, $y_{N+1}(Z)$, provides the bulk temperature of the outer stream.

RESULTS AND DISCUSSION

Equations (4) and (5) were solved for various different combinations of the governing parameters, H^*, L, K_w and quantities of practical interest evaluated according to their definitions, equations (6), for a wide range of the dimensionless axial coordinate, Z. The eigenfunction expansions were taken with a sufficiently large number of terms, N, for convergence to several digits in each situation, and the algebraic problems in equations (14) were readily solved through the appropriate software in the IMSL package [21], providing highly accurate final results.

Table 1 shows numerical results for the asymptotic Nusselt numbers both for the inner stream alone and overall, including the wall thermal resistance, according to equation (6d). As the relative wall thermal resistance is increased, as expected, the overall Nusselt number decreases due to the added resistance, but the internal convective heat transfer coefficient can either increase, for $H^* > 1$, or decrease, for $H^* < 1$. As the heat capacity flow rate ratio is increased, the formulation of the Graetz problem with the third kind boundary condition (or prescribed temperature, for $K_w = 0$) is asymptotically approached, since $d\theta_{2,av}/dZ \to 0$, according to equation (4a), or $\theta_{2,av}(Z) \to 1$. The results for $H^* = 10$ confirm such a tendency, while the results for $H^* = 1$ confirm the observation in Nunge and Gill [8], that when the flow rates and heat capacities are equal, a situation similar to a Graetz problem with prescribed uniform heat flux is recovered. In this case, the average fluid temperatures have essentially a linear variation along the axial coordinate and reproduce, although only asymptotically as will be clear in what follows, the H-problem behavior.

Table 1. Asymptotic Nusselt numbers at both the inner tube and overall for different heat capacity flow rate ratios and relative wall thermal resistances

	$Nu_i(Z\to\infty)$ / $Nu_0(Z\to\infty)$		
$H^* = \dot{m}_2C_2/\dot{m}_1C_1$	$K_w = 0$	$K_w = 0.2$	$K_w = 0.4$
1/10	10.126	7.9499 4.4289	7.1047 2.9346
1/3	5.8638	5.3540 3.4870	5.1148 2.5283
1/2	5.1109	4.8718 3.2758	4.7500 2.4358
1	4.3636	4.3636 3.0379	4.3636 2.3300
2	3.9888	4.1039 2.9097	4.1632 2.2717
3	3.8711	4.0172 2.8859	4.0954 2.2513
10	3.7150	3.8960 2.8086	4.0000 2.2222

Figure 2 presents the local Nusselt number distributions for the inner stream, within the thermal entry region, for different values of H^*, with $K_w = 0$ and $L = 1$. Also shown are the results for the Graetz problem with prescribed temperature (T-problem) and prescribed heat flux (H-problem). Clearly, the T-problem results represent a lower bound for the countercurrent double-pipe heat exchanger curves. The H-problem curve merges with the curve for $H^* = 1$ for sufficiently large axial distances, in the asymptotic region, as indicated in Table 1. However, within the thermal entry region these two curves are quite apart from each other, and the H-problem solution cannot be utilized to approximate the double-pipe heat exchanger with $H^* = 1$, as suggested in ref. [8]. This behavior could not be observed by Nunge and Gill [8] due to the relatively low order of the eigenfunction expansion employed. Also of interest is the fact that the asymptotic region is reached within a much shorter axial distance for the lower values of H^*, due to the more effective heat exchange.

The axial distributions of the interfacial wall temperature are shown in Fig. 3 for both the countercurrent and concurrent flow configurations, with $K_w = 0$, $L = 1$ and different values of H^*. It is noticeable that the uniform prescribed temperature boundary condition is approached for both flow situations as H^* increases. Also, the axial wall temperature gradients are more pronounced in the countercurrent case, especially for lower values of H^* and in the region close to the annular stream inlet. This result provides some indication that the effects of conjugated

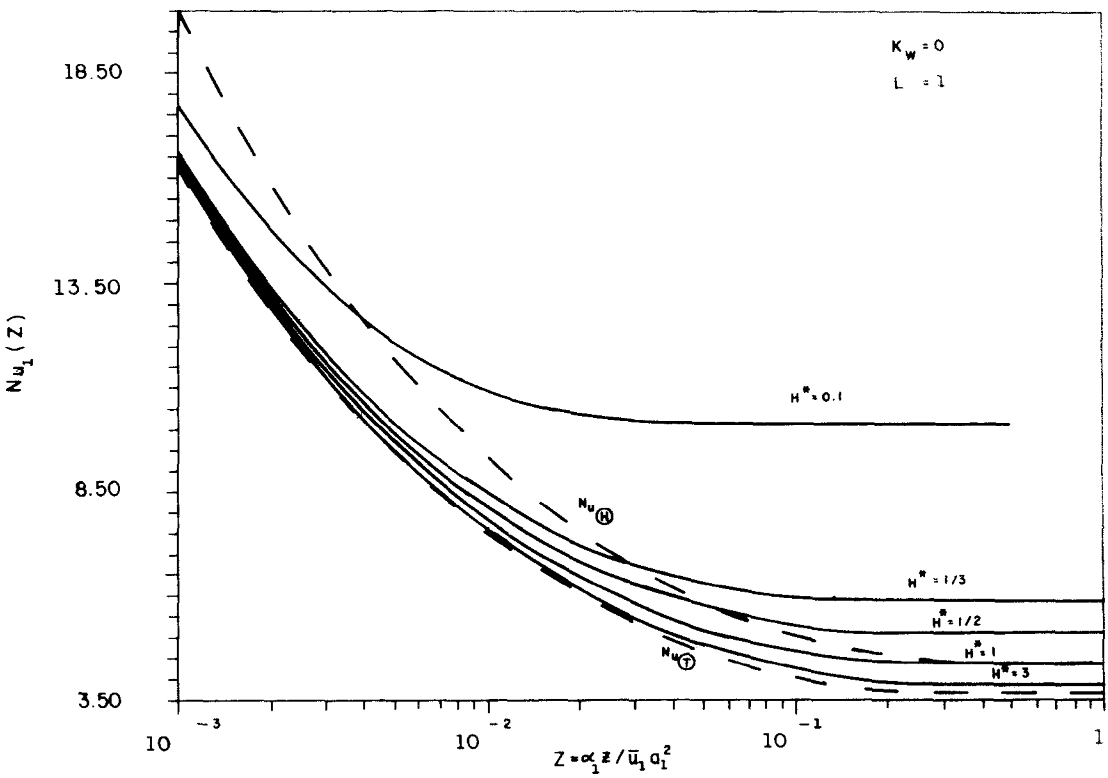

FIG. 2. Local Nusselt number at the inner tube, $Nu_1(Z)$, along the thermal entry region ($K_w = 0$; $L = 1$).

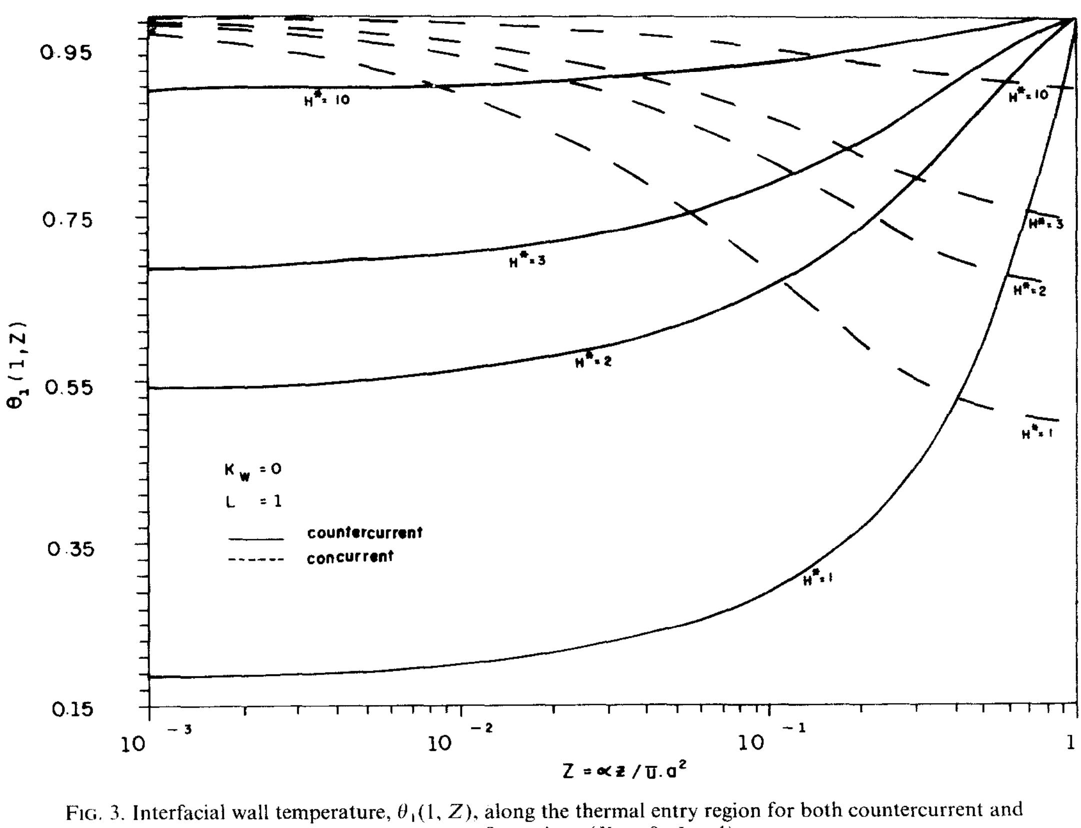

FIG. 3. Interfacial wall temperature, $\theta_1(1, Z)$, along the thermal entry region for both countercurrent and concurrent configurations ($K_w = 0$; $L = 1$).

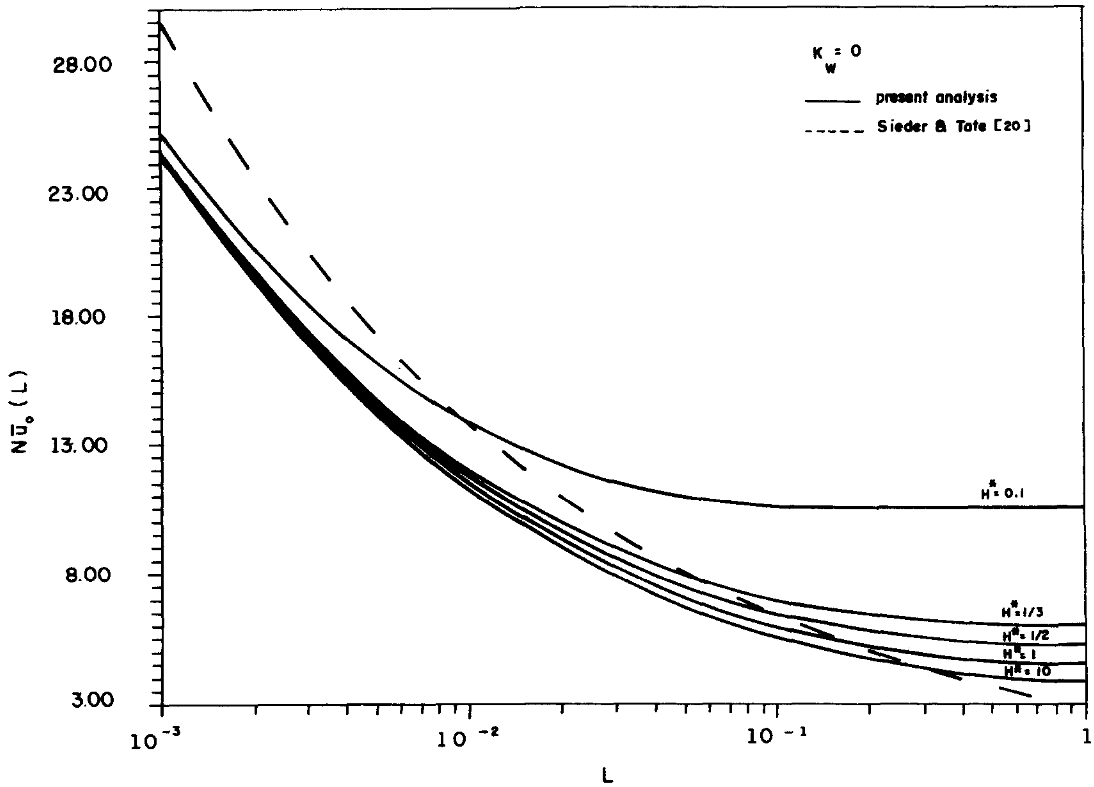

FIG. 4. Comparison of average Nusselt numbers for different heat exchanger lengths and heat capacity flow rate ratios, with the correlation of Sieder and Tate (see ref. [20]) ($K_w = 0$).

wall heat transfer must be analyzed more closely in a certain range of the parameter H^*, in order to account for longitudinal wall heat conduction [19].

In Fig. 4 we present numerical results for the average overall Nusselt number as a function of the dimensionless heat exchanger length, L, and for different values of the heat capacity flow rate ratio. Also shown is a curve representing the correlation of Sieder and Tate, recommended by Kern [20], which correlates to $\pm 12\%$ several experimental results for circular tubes and different boundary conditions. The ranges for each governing parameter covered by this expression, according to the experiments considered, are presented in ref. [8]. As already discussed by Nunge and Gill [8], this simple expression does not appropriately approach the asymptotic region, with ever decreasing Nusselt numbers for increasing exchanger length. Here, it is also apparent that for the smaller values of L, this correlation again deviates considerably from the theoretical predictions, which is probably due to the very limited range of exchanger dimensionless lengths covered by the experiments correlated, centered around the intermediate region. For larger L, the deviations are more pronounced for decreasing values of H^*.

Heat exchanger effectivenesses, for both countercurrent and concurrent configurations, are presented in Fig. 5 as a function of the dimensionless heat exchanger length and for different values of H^*. As expected, the counterflow arrangement is seen to be more effective in all cases considered, especially for $H^* < 1$, while the operation mode is not so relevant in terms of effectiveness for smaller values of L, and most noticeably for $H^* > 1$. Effectiveness charts of practical interest can be readily constructed, as briefly demonstrated by Fig. 5, without prescribing convective heat transfer coefficients.

Finally, it should be noted that the present results are expected to be applicable within the range for the relative thermal resistance of the fluids, K, recommended in ref. [12] ($K \leqslant 0.1$), since benchmark results for the counterflow situation are not available, so as to allow an inspection of the degree of approximation in the lumping procedure of the annular region for different values of K. An examination of the asymptotic results in ref. [8] confirms, to a certain extent, this expectation.

The present approach is directly applicable to turbulent flow in the outer stream and is sufficiently straightforward to be extended to handle more involved problems, such as in the cases of turbulent internal flow, wall conjugation effects and transient or periodic states. Other more complex geometries for the outer region can also be studied, provided the

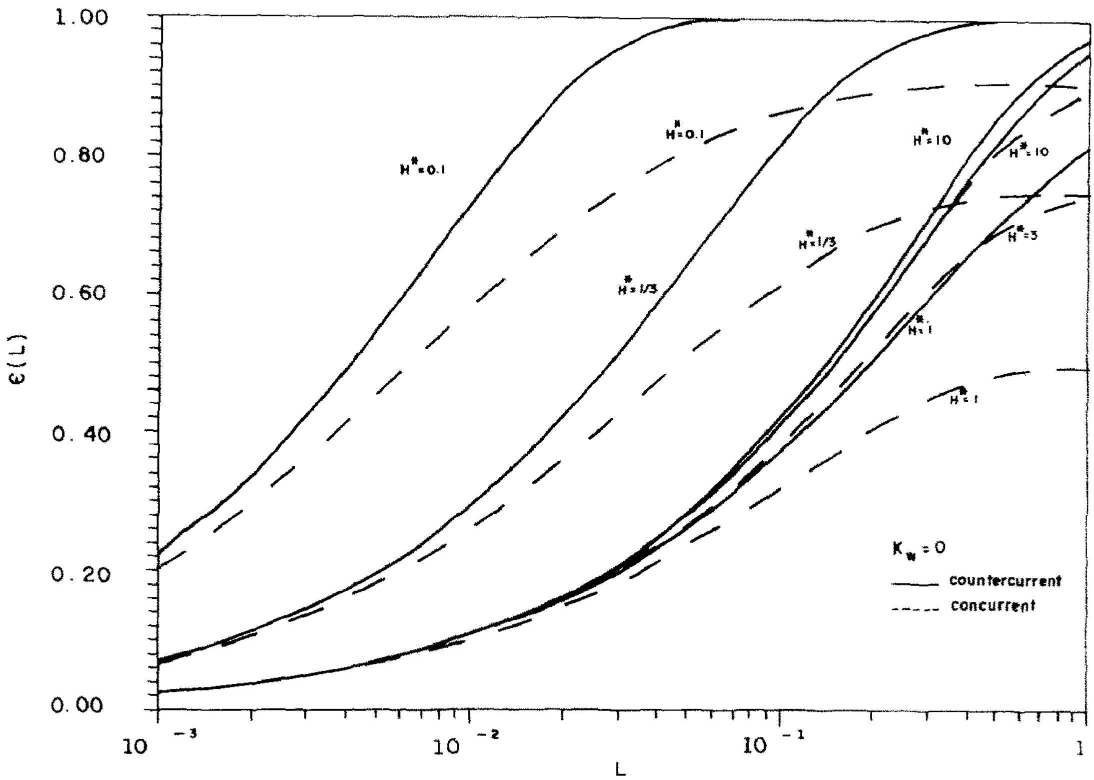

FIG. 5. Comparison of heat exchanger effectiveness between countercurrent and concurrent configurations for different exchanger lengths and heat capacity flow rate ratios ($K_w = 0$).

lumped formulation is applicable, i.e. temperature gradients can still be considered negligible in the transversal direction within the external stream.

The analysis advanced here represents an interesting alternative to purely numerical approaches, which require costly iterative procedures to obtain approximate solutions for such coupled counterflow configurations, and adds to the various classes of linear and non-linear convection–diffusion problems now tractable through the integral transform method.

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ANALYSE D'UN ECHANGEUR DE CHALEUR A DEUX TUBES EN CONTRECOURANT A L'AIDE D'UNE FORMULATION MIXTE CELLULAIRE-DIFFERENTIELLE

Résumé—On analyse les échangeurs de chaleur à deux tubes concentriques dans des conditions d'écoulements à contre-courant en développement. On emploie une formulation mixte différentielle et cellulaire en zonant radialement le champ de température dans le canal extérieur, ce qui conduit à une condition aux limites mieux impliquée pour le système différentiel interne et rend compte de la variation axiale de la température de mélange dans le canal externe. On obtient une solution analytique directe de cette classe de problèmes. Des résultats numériques pour les grandeurs thermiques sont présentés en fonction des paramètres adimensionnels actifs le long de la région d'entrée thermique, permettant des comparaisons avec la situation de co-courant, les solutions limites et les formules pratiques.

UNTERSUCHUNG EINES DOPPELROHR-GEGENSTROMWÄRMETAUSCHERS MIT HILFE EINER GEMISCHTEN KONZENTRIERT/DIFFERENTIELLEN FORMULIERUNG

Zusammenfassung—Doppelrohr-Wärmeaustauscher werden für den Fall einer thermisch nicht entwickelten Gegenströmung untersucht. Es wird eine gemischte konzentriert/differentielle Formulierung angewandt, bei der das Temperaturfeld im äußeren Kanal kreisförmig konzentriert wird. Daraus ergibt sich eine kompliziertere Randbedingung für das innere Differentialgleichungssystem, wobei eine Änderung der Temperatur im äußeren Kanal in axialer Richtung einbezogen ist. Um zu einer verläßlichen und unkomplizierten analytischen Lösung zu gelangen, wird eine allgemein gültige Integrationstechnik angewandt. Die numerischen Ergebnisse für die Wärmeübertragungsgrößen werden anhand der dimensionslosen maßgeblichen Parameter längs des thermischen Einlaufgebiets dargestellt, was einen kritischen Vergleich mit der Gegenstromsituation mit Grenzfällen sowie mit ingenieurmäßigen Korrelationen zuläßt.

АНАЛИЗ ПРОТИВОТОЧНОГО ДВУХТРУБЧАТОГО ТЕПЛООБМЕННИКА С ИСПОЛЬЗОВАНИЕМ КУСОЧНО-ПЕРЕМЕННОЙ ФОРМУЛИРОВКИ

Аннотация—Анализируются двухтрубчатые теплообменники в условиях термически развивающихся противотоков. Используется формулировка, в которой температурное поле во внешнем канале в радиальном направлении представляется в виде кусочно-переменной функции, что приводит к более сложному граничному условию для внутренней системы, содержащему аксиально изменяющуюся среднемассовую температуру внешнего канала. С целью получения надежного и прямого аналитического решения данного класса задач применяется метод обобщенных интегральных преобразований. Численные результаты для характеристик теплопереноса вдодль входного теплового участка приводятся посредством безразмерных определяющих параметров, что позволяет провести сравнение со случаем спутных потоков.